Dynamic output stabilization of control systems: an unobservable kinematic drone model

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Abstract

The problem of dynamic output stabilization is a very general and important problem in control theory. This problem is completely solved in the case where the system under consideration is uniformly observable. However, usually, nonlinear systems do not share this property: in general, systems are observable or not depending upon the control as a function of time. In this general situation, very little is known about dynamic output stabilization.
In this paper, we solve the problem for a classical academic kinematic model for drones whose observability properties are especially bad.

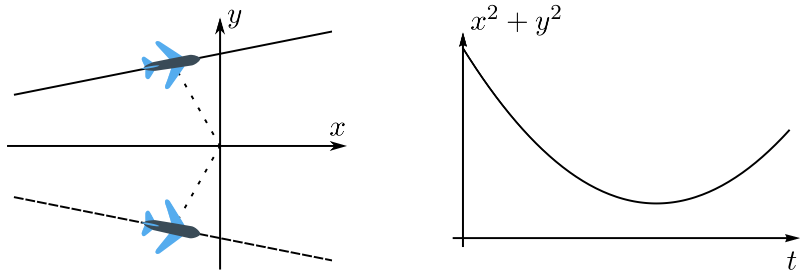
Keywords. Control systems, dynamic output stabilization, asymptotic stability

1 Introduction

Dynamic output stabilization of a dynamical system is a classical problem from control theory. If the stabilization is achievable with a state feedback law but only an output of the system is known, a natural idea is to apply this feedback to an estimation of the state provided by an observer. In the case of non-linear systems, this strategy was proved to be effective under assumption of observability for all inputs, known as uniform observability [1, 2, 3, 4, 5, 6]. In full generality, however, uniform observability is generically not satisfied [5], including for important classes such as state affine systems (bilinear dynamics with linear observation). There may exist input controls that make the system unobservable and working around them is a challenging task.

There have been attempts at building strategies for stabilization of poorly observable systems [7, 8, 9]. These approaches, however, rely on time-varying feedback. A fundamental and difficult problem is posed by the construction of time-invariant strategies relying only on an estimate of the state.

In this paper, we present a case study where a strategy for dynamic output time-invariant feedback stabilization is built around a well-studied model for drone dynamics. The aim of this paper is not reduced to proving that this system is stabilizable. Rather, it illustrates



(a) Straight trajectories (b) Corresponding output

Figure 1: Symmetries in the measured output cause some straight trajectories to be indistinguishable form each other. The plain and the dashed trajectories result in the same measurement over time.

that observers can be built to be convergent even in the presence of observability singularities in the system, and without prior knowledge of the feedback law. In [10], another example of the same kind of problem is treated with similar methodology, and the reader is invited to consult [11] where the general matter is discussed.

Our example is the following academic kinematic model of a fixed wings drone (or UAV), flying at constant altitude, with constant linear velocity:

$$\begin{cases} \dot{x} = \cos\theta, \\ \dot{y} = \sin\theta, \\ \dot{\theta} = u, \qquad -u_{\max} \leq u \leq u_{\max}. \end{cases} \tag{1}$$

This system is a variation on the revered Dubins model [12, 13, 14] where (x, y)-trajectories on the plane have a minimum possible radius of curvature $1/u_{\max}$. It has been extensively studied for the modeling of vehicles and fixed wings drones, especially in regards to trajectory optimality [15, 16, 17, 18, 19, 20].

Endowed with the only information given by $x^2 + y^2$, the square of the distance to the origin, we ask "is it possible to stabilize this system on a circular trajectory of minimal radius $1/u_{\max}$ around the origin?"

With full information, this poses no issue. However, the distance output is especially poor in this context. Indeed, under the input $u = 0$, trajectories are straight and distance measurements are indistinguishable under rotational and reflection symmetry in the plane (see Figure 1). The reflection symmetry can imply (for instance) a switch from θ to $-\theta$ that is not solvable by feedback design. Given a straight trajectory and the corresponding output, it is not possible to know if the aircraft should steer left ($u > 0$) or right ($u < 0$) to the target.

Under these very poor observability constraints, classical output stabilisation theorems cannot be applied. Nevertheless, we are able to prove the following statement.

Theorem 1 *For any smooth feedback stabilizing at the target trajectory there exists a Luenberger-type observer for system* (1)*, for which the coupled closed loop state-observer system is asymptotically stable at the target, with an arbitrarily large basin of attraction.*

Remark 1 *Knowledge of the feedback law and the desired basin of attraction may appear to be necessary for the specific choice of the Luenberger-type observer. We show in the following* *sections that the size of the basin of attraction is only dependent on the size of a positive tuning parameter. Any bounded subdomain of the plane can be covered by the basin of attraction of the coupled system if the tuning parameter is chosen large enough. Furthermore, for any choice of the parameter, local stability is satisfied. Finally, and most importantly, no assumptions on the relationship between the feedback law and the observability singularity at $u = 0$ need to be made. Regularity and stabilizability are the only necessary assumptions.*

The paper is organized as follows. In Section 2, we detail the problem and a state-affine reduction. A precise statement of our result is given and discussed. In Section 3, we present the proof of this result. In Section 4, we show some simulations and give some closing remarks and perspectives.

In the following, X' denotes the transpose of any matrix or vector X.

2 State affine formulation

2.1 State affine embedding of the problem

The issue at hand is to stabilize system (1) at the origin. But what does it mean for a drone with constant velocity? In fact, it is required that it reaches a limit motion of turning around the target achieving a circle of minimal radius $r = 1/u_{\max}$.

This leads to the following reduction of the model. We define the target set $\mathcal{T}$ by

$$\mathcal{T} = \{(x, y, \theta) \mid x = r \sin\theta, y = -r\cos\theta\}. \tag{2}$$

This set is travelled by the system under the input $u = u_{\max}$. Rather than considering stabilization to $\mathcal{T}$, a classical moving frame allows to reduce the dimension of this dynamical system and collapse $\mathcal{T}$ to a single point in the plane. We set

$$\begin{pmatrix} \tilde{x} \\ \tilde{y} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}. \tag{3}$$

In these new UAV-based coordinates $(\tilde{x}, \tilde{y}, \theta)$, system (1) can be rewritten as

$$\begin{cases} \dot{\tilde{x}} = u\,\tilde{y} + 1, \\ \dot{\tilde{y}} = -u\,\tilde{x}. \end{cases} \tag{4}$$

For a non-zero $u \in [-u_{\max}, u_{\max}]$, system (4) possesses a single equilibrium $(0, -1/u)$. In particular for $u = u_{\max}$ and $u = -u_{\max}$, they have equilibria $(0, -r)$ and $(0, r)$. They correspond to the target set $\mathcal{T}$ being browsed counter-clockwise and clockwise respectively. If u is changed for $-u$, the two equilibria are exchanged so we can indifferently consider one among the two equilibria positions for stabilization. In these new coordinates, θ does not play a role anymore. It can be preserved as an integrator of the control, but stabilization towards $\mathcal{T}$ becomes a matter of stabilization to a point in the plane.

Consider for systems (1), (4), the following *"minimum information output"*, i.e. the square distance to the origin:

$$\rho^2 = x^2 + y^2 = \tilde{x}^2 + \tilde{y}^2.$$

For $t \in [0, T]$, if $(x(t), y(t), \theta(t))$ is a trajectory of (1) for an arbitrary input control $u(t)$, it is clear that

$$(x(t)\cos\theta_0 - y(t)\sin\theta_0, x(t)\sin\theta_0 + y(t)\cos\theta_0, \theta(t) + \theta_0)$$

is a trajectory of (1) with same input and same output.

This rotational symmetry implies that system (1) is not locally weakly observable in the sense of [21]: close to any (x, y, θ), there is a continuum of points that are indistinguishable to (x, y, θ) by the observations, whatever the input $u(\cdot)$.

Thanks to the system reduction, this is not the case for system (4) (in fact it is the quotient of system (1) by the weak indistinguishability relation from [21]). However, system (4) is still not observable for all inputs: for the constant control $u \equiv 0$, knowledge of the observation ρ^2 allows reconstruction $\tilde{x}$ but $\tilde{y}$ can be reconstructed up to sign only. (This corresponds to the situation shown in Figure 1)

Remark 2 *Besides [21], one can check [22, 23, 24] for the general theory of quotienting through unobservability. The reader can refer to [11] for a brief discussion of observability singularities in the context of state-affine systems.*

The observation space of system (4) is finite-dimensional. Following [25], it can be embedded into a state-affine system. Here, we simply set $z = (z_1, z_2, z_3)$, $z_1 = \tilde{x}^2 + \tilde{y}^2$, $z_2 = \tilde{x}$, $z_3 = \tilde{y}$, and denoting the output by s, we get the bilinear system with linear observation

$$\begin{cases} \dot{z} = Az + uBz + b, \\ s = Cz, \quad u \in [-u_{\max}, u_{\max}] \end{cases} \tag{5}$$

with $A = \begin{pmatrix} 0 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$, $b = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$.

2.2 Main result

Since observation in the new state-affine system is linear, we introduce an observer $\hat{z}$ with a linear correction term in its dynamics

$$\dot{\hat{z}} = A\hat{z} + uB\hat{z} + b - K(C\hat{z} - s). \tag{6}$$

The choice of K and in general the design of the observer is open. Here we consider for system (5) a "Luenberger-type" observer, that is, with constant correction term K. For our purposes, we make the following choice for K: for some arbitrary $\alpha > 0$,

$$K' = \begin{pmatrix} \alpha & 2 & 0 \end{pmatrix}.$$

A smooth stabilizing feedback at the target for (4) is a smooth map $u : \mathbb{R}^2 \to [-u_{\max}, u_{\max}]$ such that the vector field $(u(x, y)y + 1)\partial_x - u(x, y)x\partial_y$ admits a globally asymptotically stable equilibrium at $(0, -r)$.

Stabilization of (1) can then be achieved by proving semi-global stability of the coupled closed-loop system

$$\begin{cases} \dot{\hat{z}} = A\hat{z} + u(\hat{z})\ B\hat{z} + b - KC(\hat{z} - z), \\ \dot{z} = Az + u(\hat{z})\ Bz + b, \end{cases} \tag{7}$$

in which the state z is not arbitrary, but living inside the (invariant) manifold $\mathcal{Z} = \{z \mid z_1 = z_2^2 + z_3^2\}$, and $u(\hat{z}) = u(\hat{z}_2, \hat{z}_3)$. In particular, solutions to (7) evolve in $\mathbb{R}^3 \times \mathcal{Z}$, and the target corresponds to the equilibrium (z^*, z^*) with $z^* = (r^2, -r, 0)$.

Theorem 2 *For any smooth stabilizing feedback at the target for system* (4)*, for any bounded set* $\mathcal{B}$ *in* $\mathbb{R}^3 \times \mathcal{Z}$*, there exists* α_0 *such that for all* $\alpha > \alpha_0$*, system* (7) *is asymptotically stable at* (z^*, z^*) *with basin of attraction containing* $\mathcal{B}$*.*

2.3 Discussion of the main result

Before moving on to the proof of Theorem 2, let us discuss some implications of this result.

First of all, this theorem answers the initial problem of stabilizing this kinematic drone model with distance information. It is well understood (see, for instance, [26]) that global feedback stabilization coupled with strong observability does not in general imply the possibility of global dynamical output feedback stabilization. Only semi-global stabilization should be expected.

The model is simple enough that one could design alternative approaches to this same question quite effectively. However, we are interested in the theoretical challenge of building a closed loop system for dynamic output stabilization under these very constraining observability conditions. Here, we were able to solve this problem without touching on the question of feedback design, except for regularity assumptions. Smooth stabilizing feedback laws were exhibited in [18].

The choice of the observer is free. Kalman-like observers are well adapted to time-dependent bilinear systems [27, 28, 29], and the speed of convergence can be tuned, which is useful for proving a separation principle. However strong observability assumptions are required to efficiently evaluate this convergence (see, for instance, [5, Chapter 6, Section 2]). Proving any general result of dynamic output stabilization for bilinear systems that are not strongly observable remains an open problem. Alternative strategies built on fast observers such as sliding mode are also of interest but face the same difficulties. Interestingly, a Luenberger-type observer is enough here. This points toward the possible introduction of linear matrix inequalities approaches.

3 Proof of the main result

As we explained in the previous section, our observer is a standard Luenberger-type observer for system (5), given in (6) above.

Given the geometric constraints for (7), the coupled system satisfies in error-estimate coordinates:

$$\begin{cases} \dot{\epsilon} = (A + u(\hat{z})B - KC)\epsilon, \\ \dot{\hat{z}}_2 = \hat{z}_3 u(\hat{z}) + 1 - 2C\epsilon, \\ \dot{\hat{z}}_3 = -\hat{z}_2 u(\hat{z}), \end{cases} \tag{8}$$

where ϵ is the estimation error, $\epsilon = \hat{z} - z$, and $u(\hat{z}) = u(\hat{z}_2, \hat{z}_3)$ is a stabilizing feedback law for the system

$$\begin{cases} \dot{\hat{z}}_2 = \hat{z}_3 u(\hat{z}) + 1, \\ \dot{\hat{z}}_3 = -\hat{z}_2 u(\hat{z}). \end{cases} \tag{9}$$

The question, in the remaining of this paper, is the stability of this system at $P^* \in \mathbb{R}^5$, where P^* is the point of coordinates $\{\epsilon = 0, \hat{z}_2 = 0, \hat{z}_3 = -r\}$ corresponding to the control $u = u_{\max} = 1/r$.

There are three steps to the proof of the (semi) global asymptotic stability of this coupled system (8): proof of local asymptotic stability, proof that bounded trajectories go to the target, proof that all trajectories starting in a given compact set are bounded.

3.1 Local asymptotic stability

We follow a classical scheme of proof. At the target point $\{\epsilon = 0, \hat{z}_2 = 0, \hat{z}_3 = -r\}$ the linearized system is lower triangular.

Since $KC = A' + \alpha e_{11}$, the linearization relative to the ϵ-part of the system is given by $\dot{\epsilon} = (A - A' + u_{\max} B - \alpha e_{11})\epsilon$ where e_{11} denotes the 3×3 matrix with coefficient in position $(1, 1)$ set to 1, and others to 0. Then $\|\epsilon\|^2$ is a Lyapunov function for the sub-system, as

$$\frac{1}{2}\frac{d}{dt}\|\epsilon\|^2 = \epsilon' A\epsilon - \epsilon' KC\epsilon = -\alpha\epsilon_1^2. \tag{10}$$

Since the pair $(C, (A-A'+u_{\max}B-\alpha e_{11}))$ is observable, this implies that 0 is an asymptotically stable equilibrium by LaSalle's theorem. In particular, this implies that the eigenvalues relative to the ϵ-part all have negative real part.

Notice that the $\hat{z}$-diagonal block of the linearization of system (8) (i.e. forgetting about ϵ) coincides with the linearization of the system (9). Since u is a feedback law stabilizing (9) at $(0, -r)$, its linearized can only have eigenvalues of non-positive real part.

Furthermore, the asymptotic stability of (9) implies the existence of a center manifold for (9) at $(0, -r)$ that we denote $\mathcal{C}$ (possibly empty if both eigenvalues of the linearized system have strictly negative real part). If $\mathcal{C}$ is nonempty, $\mathcal{C}$ is an invariant manifold inside the invariant manifold $\{\epsilon = 0\}$ for the coupled system (8). Since all other eigenvalues have strictly negative real part, $\mathcal{C}$ is then also a (stable) center manifold for the full coupled system (8). Hence we conclude at the asymptotic stability of the system at P^*.

3.2 Bounded trajectories converge to the target

First, along a bounded trajectory $(\epsilon(t), \hat{z}(t))$ of system (8), $C\epsilon(t)$ tends to zero. Indeed (10) holds. Therefore, $C\epsilon(t) = \epsilon_1(t)$ is a $\mathbb{L}^2$ function over $\mathbb{R}+$. Moreover, $C\epsilon(t)$ has bounded derivative since $\dot{\epsilon_1} = -\alpha\epsilon_1 + 2\epsilon_2$ and we are considering a bounded trajectory. A $\mathbb{L}^2$ function with bounded derivative tends to zero.

Looking at the $\hat{z}$-equation in (8), we see that in the ω-limit set Ω of the trajectory $(\epsilon(t), \hat{z}_2(t), \hat{z}_3(t))$, the $(\hat{z}_2, \hat{z}_3)$ part of the system follows (9) again. This is the equation of the feedback system, which is globally asymptotically stable by assumption. Hence, by the general fact of invariance and closure of the ω-limit set, Ω contains at least one trajectory such that $\hat{z}_2 \equiv 0, \hat{z}_3 \equiv r$.

Now, plugging $u(\hat{z}) = -u_{\max}$ in the equation of ϵ, we see that $C\epsilon \equiv 0$ (preserved along trajectories in Ω) can only be achieved if $\epsilon \equiv 0$ by observability of the system for $-u_{\max}$. Thus $(0, (0, r))$ belongs to Ω and, therefore, the trajectory $(\epsilon(t), \hat{z}(t))$ enters in finite time in the basin of attraction of P^*.

3.3 All semi-trajectories are bounded

As shown in Section 3.2, we have (10). This implies that ϵ is bounded and $C\epsilon(t) = \epsilon_1(t)$ tends to zero. But, $\dot{\epsilon}_1 = -\alpha\epsilon_1 + 2\epsilon_2$. Hence

$$\epsilon_1(t) = e^{-\alpha t}\epsilon_1(0) + 2\int_0^t e^{-\alpha(t-s)}\epsilon_2(s)ds$$

and

$$|\epsilon_1(t)| \leq e^{-\alpha t}|\epsilon_1(0)| + \frac{2\|\epsilon(0)\|}{\alpha}(1 - e^{-\alpha t}).$$

Therefore, we have the following.

Lemma 1 *For all $\psi > 0$, $\tau > 0$ and $\eta > 0$ there exists $\alpha_\tau > 0$ such that any semi-trajectory $(\epsilon, \hat{z})$ of (8) with $\alpha > \alpha_\tau$ and $\|\epsilon(0)\| < \psi$ has $|\epsilon_1(t)| < \eta$ for all $t > \tau$.*

Let V be a strict proper Lyapunov function for the feedback system (9). Such a Lyapunov function can be obtained by applying inverse Lyapunov's theorems (see, for instance, [30, 31]).

Let K be any compact subset of $\mathbb{R}^5$ (the state space of (8)) and let $\Pi : \mathbb{R}^5 \to \mathbb{R}^2$ be the projection on the two last components $(\hat{z}_2, \hat{z}_3)$ (that are the estimates of $(\tilde{x}_1, \tilde{x}_2)$ in (4)). Let $k \in \mathbb{N}$ be a large enough integer such that $\Pi(K) \subset D_k$, where we denote the level sets of V by

$$D_\delta = \{(\tilde{x}_1, \tilde{x}_2) \mid V(\tilde{x}_1, \tilde{x}_2) \leq \delta\}.$$

Let $\psi > 0$ be such that $K \subset [-\psi, \psi]^3 \times D_k$. Notice that the vector field $(u(x, y)y + 1 - 2\epsilon_1)\partial_x - u(x, y)x\partial_y$ is uniformly bounded with respect to $|\epsilon_1| \leq \psi$ on D_{k+1}. We denote by $R > 0$ a uniform bound on the norm of the vector field. Then by setting

$$\tau = \frac{1}{R+1}\text{dist}(D_k, D_{k+1}) > 0,$$

we have that if ζ is a semi-trajectory of (8) starting in K, then $\Pi(\zeta(t))$ remains in the interior of D_{k+1} for all $t \in [0, \tau]$, since the norm R of the velocity vector $(\dot{\tilde{x}}_1, \dot{\tilde{x}}_2)$ is small enough for that.

This fact is independent on the choice of α since the bound R is uniform and ϵ_1 is decreasing.

Denoting $f(x, y) = (u(x, y)y + 1)\partial_x - u(x, y)x\partial_y$ and L_f the Lie-derivative with respect to the vector field f, let

$$m = \inf_{D_{k+1}\setminus D_k} |L_f V| > 0.$$

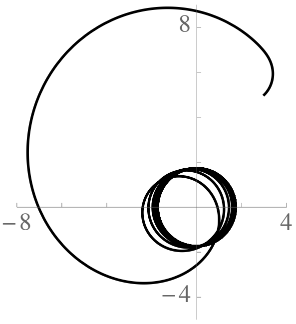
As a consequence of Lemma (1), we can choose $\alpha > 0$ such that any semi-trajectory ζ of (8) with $\zeta(0) \in K$, satisfies

$$2|\epsilon_1(t)| \sup_{D_{k+1}\setminus D_k} \left|\frac{\partial V}{\partial \tilde{x}_1}\right| < m, \qquad \forall t > \tau.$$

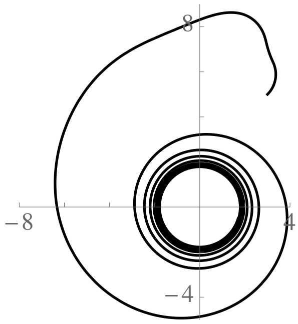
This implies that if $\Pi(\zeta(t)) \in D_{k+1} \setminus D_k$ at $t > \tau$, then $\frac{d}{dt}V(\hat{z}_2, \hat{z}_3) < 0$.

However, if there exists $t > 0$ such that $\Pi(\zeta(t)) \notin D_{k+1}$, this implies the existence of a time $t' \in (\tau, t)$ such that $\Pi(\zeta(t')) \in D_{k+1}$ and

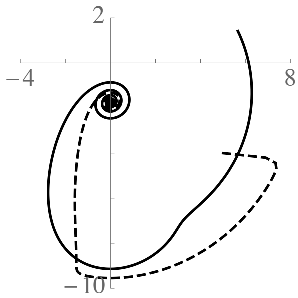
$$\frac{d}{dt}V(\hat{z}_2, \hat{z}_3)_{|t=t'} > 0,$$



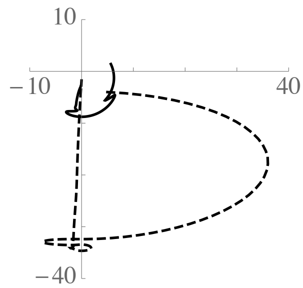
(a) Original coordinates



(b) Original coordinates



(c) Moving frame



(d) Moving frame

Figure 2: Simulations of the output feedback stabilization strategy, in both original and moving frame coordinates. The left column corresponds to $\alpha = 30$ and the right column to $\alpha = 0.5$. The plain curve corresponds to the state of the system while the dashed curve corresponds to the observer.

which we excluded. Therefore, $(\hat{z}_2, \hat{z}_3)$ remains in D_{k+1} for ever. We already know that $\epsilon(t)$ is bounded. Hence, the full trajectory is bounded.

This ends the proof of Theorem 2.

4 Conclusion and perspectives

We show two simulations in which we used the smooth stabilizing feedback control law from [18, Theorem 2.2] relative to system (4). The first simulation is with large α ($\alpha = 30$), the second with small α ($\alpha = 0.5$), see Figure 2. Both are taken with initial conditions $(x_0, y_0, \theta_0) = (3, 5, \pi/4)$ and $\hat{z}_0 = (-7, 5, 4)$. On the top line of the figure are represented the trajectories of the drone (1) in the (x, y)-plane. On the bottom are represented the trajectories of the reduced system (4) together with the corresponding observer state estimate. The strong inobservability value $u = 0$ is actually crossed, at inflexion points of the trajectory.

To finish, we would like to point out the challenge presented by practical output stabilization of control systems in the unobservable case. Here we have treated a case where the target point is an observable point, and showed that after state-affine embedding, a Luenberger-type observer was converging reliably enough to allow a separation principle to take place. We refer to [10] for a case where the target control makes the system unobservable. It is particularly challenging to consider, for unobservable bilinear or bilinearizable systems, the coupling of a stabilizing feedback law with a Kalman-type observer. Such a result would be very important.

For what regards this particular kinematic drone model, since straight trajectories can be time optimal, we consider the coupling of the observability problem with the minimum time optimal synthesis a particularly interesting question worthy of further research.

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